EEL 301 MINOR TEST 1

Duration: 1 hour | Total Marks: 20

Question 1. *(8 marks)* Starting from the definition of the Laplace Transform, prove the Final Value theorem: $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$, if $\lim_{t\to\infty} f(t)$ exists. Clearly explain how the condition 'if $\lim_{t\to\infty} f(t)$ exists' is used in the proof? Illustrate your explanation with examples.

Question 2. *(6 marks)* A dc-dc converter is a device that takes as an input an unregulated dc voltage and provides a regulated dc voltage as its output. The output voltage may be lower (buck converter), higher (boost converter), or the same as the input voltage. Switching dc-dc converters have a semi-conductor active switch (BJT or FET) that is closed periodically with a duty cycle d in a pulse width modulate (PWM) manner. For a boost converter, averaging techniques can be used to arrive at the following state equations:

$$\begin{aligned} L\frac{di_L}{dt} &= -(1-d)v_C + E_s, \\ C\frac{dv_C}{dt} &= (1-d)i_L - \frac{v_C}{R}, \end{aligned}$$

where L and C are respectively the values of internal inductance and capacitance; i_L is the current through the internal inductor; R is the resistive load connected to the converter; E_s is the dc input voltage; and the capacitor voltage, v_C, is the converter's output.

(a) Write the converter's equations in the form $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, $\mathbf{y} = \mathbf{Cx}$.

(b) Using the **A**, **B**, and **C** matrices of Part (a), obtain the converter's transfer function $V_C(s)/E_s(s)$.

Question 3. *(6 marks)* Each inner ear in a human has a set of three nearly perpendicular semicircular canals of about $0.28mm$ in diameter filled with fluid. Hair-cell transducers that deflect with skull movements and whose main purpose is to work as attitude sensors as well as help us maintain our sense of direction and equilibrium are attached to the canals. As the hair cells move, they deflect a waterproof flap called the *cupula*. It has been shown that the skull and cupola movements are related by the following equation:

$$J\ddot{\phi} + b\dot{\phi} + k\phi = (aJ)\ddot{\psi},$$

where,

- J = moment of inertia of the fluid in the thin tube (constant),
- b = torque per unit relative angular velocity (constant),
- k = torque per unit relative angular displacement (constant),

- a = constant (constant),
- $\phi(t)$ = angular deflection of the *cupula* (output),
- $\ddot{\psi}(t)$ = skull's angular acceleration (input).

Find the transfer function $\Phi(s)/\ddot{\Psi}(s)$. For different values of the constants, sketch the different possible output responses to a step input?